東北大学機関リポジトリ
TOUR
Tohoku University Repository

On the Motion of the Surface of the Sea due to a Submarine Earthquake

著者	Honda Hirokichi, Nakamura Kohei
雑誌名	Science reports of the Tohoku University. Ser. 5, Geophysics
巻	7
号	1
ページ	17-34
発行年	1955-08
URL	http://hdl.handle.net/10097/44533

On the Motion of the Surface of the Sea due to a Submarine Earthquake

By HIROKICHI HONDA and KÔHEI NAKAMURA
Geophysical Institute, Faculty of Science, Tôhoku University

(Received 10 May 1955)

Abstract

The theory of the motion of the surface of the sea of uniform depth, due to the dilatational and distortional waves emitted from an impulsive point source within the elastic solid bottom, is developed. The results are applicable for the region not very far from the epicenter.

1 Introduction

The vesseles on the surface of the sea are sometimes shaken violently up and down, when a shallow focus submarine earthquake occurs in the nearby region. The phenomenon is called the "sea shocks". It was reported for instance that the steamship Heian-Maru was shaken very severely during about 5 minutes from $17^h\ 31^m$ to 36^m (G.M.T.) on Mar. 2, 1933, at a point about 500 km distant from the epicenter of the great Sanriku Earthquake. The dilatational (P) and distortional (S) waves should arrive there at $17^h\ 32.1^m$ and 33.2^m respectively, the time of occurrence of the earthquake being $17^h\ 30^m\ 54^s$. It has been considered generally that the P and S waves emitted from the hypocenter of a shallow focus submarine earthquake are refracted at the sea bottom, and the elastic (sound) waves transmitted into the sea to cause the sea shocks. The thorough theoretical investigations on the phenomenon, however, seem not to have been presented.

The normal mode propagation of sound from a point source in a liquid layer overlying a liquid bottom was investigated by C. L. PEKERIS (1948), and the case of solid bottom by F. PRESS and M. EWING (1948, 1950). The importance of the stationary value of the group velocity for the elastic waves transmitted across the acoustic system was pointed out in these papers. C. B. OFFICER (1953) evaluated the branch line integrals for the case treated by PEKERIS. PRESS, EWING and I. TOLSTOY (1950) investigated the motion of the Airy phase at the surface of the solid bottom when the point source is situated at some depth within the bottom. Almost all these researches have been confined to the region very far from the epicenter, where the normal mode wave motions predominate. HONDA and NAKAMURA (1952, 1954) studied the propagation of sound from a point source in the sea overlying a solid bottom, in the region near the epicenter, and treated the waves reflected at the surface of the sea and reflected and refracted at the surface of the bottom.

In the present paper, the authors intend to investigate theoretically the motion of the surface of the sea of uniform depth overlying a solid bottom, due to the P and S pulses emitted from a point source within the solid bottom. We take the particular use of the method of the steepest descent, to derive the solutions from which the motion of the sea surface due to the sound waves refracted into the sea and reflected at the surface of the sea or the bottom can be obtained. The motions of the sea surface due to the refracted and reflected sound waves predominate in the region not very far from the epicenter, and those due to the normal mode propagation predominate at large distances from the epicenter. We will take no account of the normal mode propagation of the waves here, as it has been fully discussed by PEKERIS and PRESS et al. The results of our present calculation may be applicable for the region not very far from the epicenter, and may be utilized for the consideration of the sea shocks caused by a submarine earthquake.

2 Theory

Let us consider the propagation of the elastic waves through the system consisting of a liquid layer of thickness H, density ρ_1, compressional wave velocity v_1, and incompressibility λ_1, overlying a semi-infinite elastic solid of density ρ_2, compressional wave velocity $v_{2,p}$ distortional wave velocity $v_{2,s}$ and Lamé's constants λ_2, μ_2. The subscripts 1 and 2 refer to the liquid layer and the bottom respectively. The x and y axes are taken in the surface of the liquid layer, and the z axis is directed vertically downward. Assuming symmetry about the z axis, we put $r=\sqrt{x^2+y^2}$. The radial and the vertical components of displacement are denoted by δ_r and δ_z respectively.

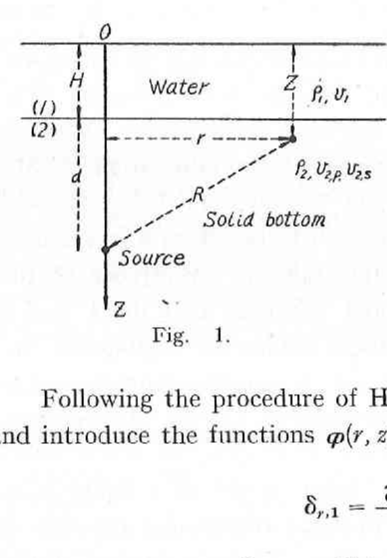

Fig. 1.

Following the procedure of H. LAMB (1904), we assume simple harmonic motion and introduce the functions $\varphi(r, z)$ and $\psi(r, z)$ defined by the equations:

$$\delta_{r,1}=\frac{\partial\varphi_1}{\partial r}, \quad \delta_{z,1}=\frac{\partial\varphi_1}{\partial z}, \tag{1}$$

$$\delta_{r,2}=\frac{\partial\varphi_2}{\partial r}+\frac{\partial^2\psi_2}{\partial r\partial z}, \quad \delta_{z,2}=\frac{\partial\varphi_2}{\partial z}+\frac{\partial^2\psi_2}{\partial z^2}+k_{2,s}{}^2\psi_2, \tag{2}$$

where

$$(\nabla^2+k_1{}^2)\,\varphi_1=0, \quad (\nabla^2+k_{2,p}{}^2)\,\varphi_2=0, \quad (\nabla^2+k_{2,s}{}^2)\,\psi_2=0, \tag{3}$$

$$k_1=\frac{\omega}{v_1}, \quad k_{2,p}=\frac{\omega}{v_{2,p}}, \quad k_{2,s}=\frac{\omega}{v_{2,s}}, \quad \nabla^2=\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{\partial^2}{\partial z^2}. \tag{4}$$

The time factor $\exp(i\omega t)$ is here and often hereafter temporarily omitted. The vertical stress p_{zz} at the surface of the sea and the tangential stress p_{zr} at the interface must be zero, and the vertical stress and the vertical displacement have to be continuous at the interface. When the periodic compressional waves are supposed to be emitted from a point source ($r=0$, $z=H+d$) within the solid bottom, its displacement potential

φ is denoted as

$$\varphi = \exp(-ik_{2,p}R)/R, \quad \text{where} \quad R = \sqrt{r^2+(H+d-z)^2} \tag{5}$$

and the potentials φ_1, φ_2 and ψ_2 are assumed to be of the form :

$$\varphi_1 = \int_0^\infty A \sin(\xi z)\, J_0(kr)dk, \quad \text{for} \quad 0 \leqq z \leqq H, \tag{6}$$

$$\varphi_2 = \int_0^\infty B \exp(-\eta z)\, J_0(kr)dk + \frac{\exp(ik_{2,p}R)}{R} + \frac{\exp(-ik_{2,p}R')}{R'}, \text{ for } H \leqq z, \tag{7}$$

$$\psi_2 = \int_0^\infty C \exp(-\zeta z)\, J_0(kr)dk, \qquad \text{for} \quad H \leqq z \tag{8}$$

where

$$\xi^2 = k_1^2 - k^2, \quad \eta^2 = k^2 - k_{2,p}^2, \quad \zeta^2 = k^2 - k_{2,s}^2, \quad R' = \sqrt{r^2+(z-H+d)^2}, \tag{9}$$

$$Re(\xi) > 0, \quad Re(\eta) > 0, \quad Re(\zeta) > 0. \tag{10}$$

PRESS and EWING (1948, 1950) showed that φ_1 to be

$$\varphi_1 = -2\, k_{2,s}^2 \int_0^\infty \frac{(2\, k^2 - k_{2,s}^2)\sin(\xi z)}{\xi F(k)\cos(\xi H)} \exp(-\eta d)\, J_0(kr)kdk, \tag{11}$$

$$F(k) = \frac{\rho_1 \eta}{\rho_2 \xi} k_{2,s}^4 \tan(\xi H) - \left[4\, k^2 \eta\zeta - (2\, k^2 - k_{2,s}^2)^2\right]. \tag{12}$$

If the medium is extended infinitely, the displacement due to the potential φ occurs only in the radial direction, in the spherical coordinates whose origin being at the hypocenter, and its radial component δ_R is

$$\delta_R = -ik_{2,p} \exp(-ik_{2,p}R)/R, \quad \text{when} \quad k_{2,p}R \gg 1. \tag{13}$$

When the periodic distortional waves of a certain type are assumed to be emitted from the same point source, and its potenital ψ is expressed as

$$\psi = \exp(-ik_{2,s}R)/R, \tag{14}$$

the potentials φ_1', φ_2' and ψ_2' are assumed to be of the form :

$$\varphi_1' = \int_0^\infty A' \sin(\xi z)\, J_0(kr)dk, \quad \text{for} \quad 0 \leqq z \leqq H, \tag{15}$$

$$\varphi_2' = \int_0^\infty B' \exp(-\eta z)\, J_0(kr)dk, \quad \text{for} \quad H \leqq z, \tag{16}$$

$$\psi_2' = \int_0^\infty C' \exp(-\zeta z)\, J_0(kr)dk + \frac{\exp(-ik_{2,s}R)}{R} - \frac{\exp(-ik_{2,s}R')}{R'}, \quad \text{for} \quad H \leqq z. \tag{17}$$

It can be shown that φ_1' to be

$$\varphi_1' = -4\, k_{2,s}^2 \int_0^\infty \frac{\eta \sin(\xi z)}{\xi F(k)\cos(\xi H)} \exp(-\zeta d)\, J_0(kr)k^3 dk. \tag{18}$$

If the medium is extended infinitely, the displacement due to the potential ψ occurs only in the direction transverse to the direction of propagation in the plane containing the z axis, and the transverse component δ_ψ is

$$\delta_\psi = -k_{2,s}{}^2 \sin\chi \exp(-k_{2,s}R)/R\,, \tag{19}$$

where $k_{2,s}R \gg 1$, $x = R\sin\chi\cos\tilde{\omega}$, $y = R\sin\chi\sin\tilde{\omega}$, $z = R\cos\chi$, the motion being independent of $\tilde{\omega}$.

3 The Motion of the Surface of the Sea due to the P Waves emitted from a Point Source beneath the Bottom

(i) The displacement components δ_r and δ_z of the motion of the surface of the sea due to the P waves emitted from a point source beneath the bottom, are obtained from (1) and (11) to be

$$\delta_r = 0\,,$$

$$\delta_z = -2\,k_{2,s}{}^2 \int_0^\infty \frac{2\,k^2 - k_{2,s}{}^2}{F(k)\cos(\xi H)} \exp(-\eta d)\, J_0(kr) k dk\,. \tag{20}$$

The motion of the sea surface occurs only in the vertical direction.

Let us put

$$K = \frac{i\left[4k^2\eta\zeta - (2k^2 - k_{2,s}{}^2)^2\right] + \dfrac{\rho_1}{\rho_2}\dfrac{\eta}{\xi}k_{2,s}{}^4}{i\left[4k^2\eta\zeta - (2k^2 - k_{2,s}{}^2)^2\right] - \dfrac{\rho_1}{\rho_2}\dfrac{\eta}{\xi}k_{2,s}{}^4}\,, \tag{21}$$

and assume that the integrand of (20) can be expanded as following:

$$\delta_z = -4\,ik_{2,s}{}^2 \int_0^\infty M \exp(-i\xi H)\left[1 - K\exp(-2\,i\xi H) + K^2\exp(-4\,i\xi H) - \cdots\cdots\right]\exp(-\eta d)\, J_0(kr)\, k dk\,, \tag{22}$$

where

$$M = \frac{2\,k^2 - k_{2,s}{}^2}{i\left[4\,k^2\eta\zeta - (2\,k^2 - k_{2,s}{}^2)^2\right] - \dfrac{\rho_1}{\rho_2}\dfrac{\eta}{\xi}k_{2,s}{}^4}\,.$$

δ_z can be expressed as

$$\delta_z = \delta_{z,0} + \delta_{z,1} + \delta_{z,2} + \cdots\cdots\cdots, \tag{23}$$

where

$$\delta_{z,0} = -4\,ik_{2,s}{}^2 \int_0^\infty M \exp(-i\xi H)\exp(-\eta d)\, J_0(kr) k dk\,, \tag{24}$$

$$\delta_{z,1} = 4\,ik_{2,s}{}^2 \int_0^\infty M \exp(-3\,i\xi H)\, K \exp(-\eta d)\, J_0(kr) k dk\,, \tag{25}$$

$$\delta_{z,2} = -4ik_{2,s}{}^2 \int_0^\infty M \exp(-5\,i\xi H) K^2 \exp(-\eta d)\, J_0(kr) k dk\,. \tag{26}$$

Now we will evaluate the integral (24) at first. We assume that $\lambda_2 = \mu_2$, i.e. $v_{2,p} = \sqrt{3}\, v_{2,s}$, and $v_1 < v_{2,s}$, and k and w to be complex quantities. Putting

$$k = k_{2,p}\sin w\,, \tag{27}$$

$$v_1/v_{2,p} = k_{2,p}/k_1 = n < \frac{1}{\sqrt{3}}\,, \quad \rho_2/\rho_1 = m\,, \tag{28}$$

we have

$$\xi = k_{2,p}\sqrt{\frac{1}{n^2} - \sin^2 w}\,, \quad \eta = ik_{2,p}\cos w, \quad \zeta = ik_{2,p}\sqrt{\cos^2 w + 2}\,, \tag{29}$$

$$Im\sqrt{\frac{1}{n^2} - \sin^2 w} < 0\,, \; Im\,(\cos w) < 0\,, \; Im\sqrt{\cos^2 w + 2} < 0\,. \tag{30}$$

Remembering the relations $2J_0(z) = H_0^{(1)}(z) + H_0^{(2)}(z)$, and $H_0^{(1)}(-z) = -H_0^{(2)}(z)$, $\delta_{z,0}$ can be expressed as

$$\delta_{z,0} = -2k_{2,s}^2\int_{-\pi/2-i\infty}^{\pi/2+i\infty} N\sin w\exp\Big[-ik_{2,p}(d\cos w + H\sqrt{\frac{1}{n^2} - \sin^2 w})\Big]H_0^{(2)}(k_{2,p}\sin w\cdot r)dw, \tag{31}$$

$$N = \frac{m\cos w(2\cos^2 w+1)\sqrt{\frac{1}{n^2} - \sin^2 w}}{m\sqrt{\frac{1}{n^2} - \sin^2 w}\,\big[(2\cos^2 w+1)^2 + 4\sin^2 w\cos w\sqrt{\cos^2 w+2}\,\big] + 9\cos w}\,.$$

Using the asymptotic expansion

$$H_0^{(2)}(z) \sim \sqrt{\frac{2}{\pi z}}\exp\Big(-iz + i\frac{\pi}{4}\Big), \tag{32}$$

we have

$$\delta_{z,0} = -2\,k_{2,s}^2\sqrt{\frac{2}{\pi\,k_{2,p}r}}\exp\Big(i\frac{\pi}{4}\Big)\int_{-\pi/2-i\infty}^{\pi/2+i\infty} N\sqrt{\sin w}\;\times$$

$$\exp\Big[-ik_{2,p}\Big(r\sin w + d\cos w + H\sqrt{\frac{1}{n^2} - \sin^2 w}\,\Big)\Big]dw\,. \tag{33}$$

The factor K is expressed by

$$K = \frac{m\sqrt{\frac{1}{n^2} - \sin^2 w}\,\big[(2\cos^2 w+1)^2 + 4\sin^2 w\cos w\sqrt{\cos^2 w+2}\,\big] - 9\cos w}{m\sqrt{\frac{1}{n^2} - \sin^2 w}\,\big[(2\cos^2 w+1)^2 + 4\sin^2 w\cos w\sqrt{\cos^2 w+2}\,\big] + 9\cos w}\,. \tag{34}$$

(ii) The integrand of (33) contains the two valued functions $\lambda_w = \sqrt{1/n^2 - \sin^2 w}$ and $\lambda_s = \sqrt{\cos^2 w + 2}$, and four leaved Riemann surface is needed for their representation. The integrand has also infinitely many poles, and the residues give the normal mode solutions, but these are taken out of account in this paper as has been stated.

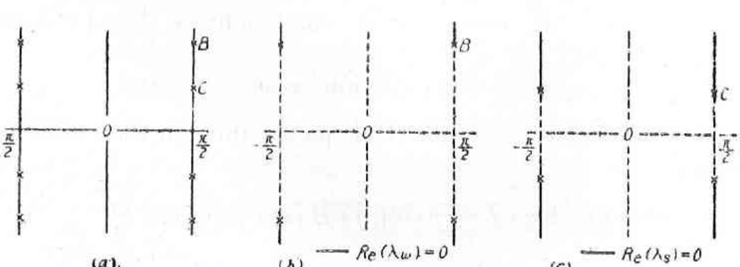

Fig. 2. w-plane

We confine ourselves to the region of w-plane bounded by two straight lines $Re(w)=\pm\pi/2$ and a certain region adjoining it, if necessary. The points B($\cos w=-i\sqrt{1/n^2-1}$, $\sin w=1/n$), C ($\cos w=-i\sqrt{2}$, $\sin w=\sqrt{3}$) &. c. in Fig. 2a, are the branch points on w-plane. The lines along which $Re(\lambda_w)=0$ and $Re(\lambda_s)=0$ are shown by the solid lines and those along which $Im\ (\lambda_w)=0$ and $Im\ (\lambda_s)=0$ by the broken lines in Fig. 2b and Fig. 2c respectively. We assume that the cuts along which the four leaves coalesce, are given by $Re(\lambda_w)=0$ and $Re(\lambda_s)=0$. On one of the four sheets on which $Re(\lambda_w)>0$ and $Re(\lambda_s)>0$, we take the given path of integration $\mathrm{L}(-\pi/2-i\infty\rightarrow\pi/2+i\infty)$. The conditions $Im(\gamma)<0$, $Im(\lambda_w)<0$ and $Im(\lambda_s)<0$ are fulfilled along L.

When the wave lengths of the waves concerned are assumed to be small compared with d, H and r, we can adopt the method of the steepest descent for the evaluation of the integral (33).

Putting

$$f(w)=-ik_{2,p}\left[r\sin w+d\cos w+H\sqrt{\frac{1}{n^2}-\sin^2 w}\,\right], \tag{35}$$

we have

$$\frac{\partial f(w)}{\partial w}=-ik_{2,p}\left[r\cos w-d\sin w-H\,\frac{\sin w\cos w}{\sqrt{\dfrac{1}{n^2}-\sin^2 w}}\right]. \tag{36}$$

As $f'(w)$ vanishes at $w=\psi_0$, where

$$r=R_0\sin\psi_0\,,\quad \frac{H\cos\psi_0}{\sqrt{\dfrac{1}{n^2}-\sin^2\psi_0}}+d=R_0\cos\psi_0\,, \tag{37}$$

$w=\psi_0$ is the saddle point. We have

$$\left[\frac{\partial^2 f(w)}{\partial w^2}\right]_{w=\psi_0}=ik_{2,p}\left[\frac{d}{\cos\psi_0}+\frac{H\cos^2\psi_0}{\sqrt{\dfrac{1}{n^2}-\sin^2\psi_0}}+\frac{H\sin^2\psi_0\cos^2\psi_0}{\left(\dfrac{1}{n^2}-\sin^2\psi_0\right)^{3/2}}\right]$$

$$=ik_{2,p}\left[R_0-\frac{H\left(\dfrac{1}{n^2}-1\right)\sin^2\psi_0}{\left(\dfrac{1}{n^2}-\sin^2\psi_0\right)^{3/2}}\right]. \tag{38}$$

Putting $w=p+iq$, we have

$$\sqrt{\frac{1}{n^2}-\sin^2 w}=(\alpha^2+\beta^2)^{1/4}\left(\cos\frac{\varphi}{2}-i\sin\frac{\varphi}{2}\right),$$

$$\alpha=\cos^2 p\cosh^2 q-\sin^2 p\sinh^2 q+\frac{1}{n^2}-1=E\cos\varphi\,,$$

$$\beta=2\sin p\cos p\sinh q\cosh q=E\sin\varphi\,. \tag{39}$$

The path of the steepest descent L' which passes through the saddle point ($p=\psi_0$, $q=0$) is given by

$$r\sin p\cosh q+d\cos p\cosh q+H(\alpha^2+\beta^2)^{1/4}\cos\frac{\varphi}{2}$$

$$=r\sin\psi_0+d\cos\psi_0+H\sqrt{\frac{1}{n^2}-\sin^2\psi_0}\,. \tag{40}$$

In the limit $q \to \pm\infty$, we see that $p \to \pm\pi/2+\psi_0'$ along L′, where $\tan\psi_0'=r/(d+H)$. L′ extends from $-\pi/2+\psi_0'-i\infty$ to $\pi/2+\psi_0'+i\infty$, and makes the angle $\pi/4$ with the real axis at the saddle point, as $Im[f''(\psi_0)]>0$ there, or as can be shown directly by the use of (40). The paths of the steepest descent passing through the branch points B and C are shown similarly to pass away towards $\pi/2+\psi_0'+i\infty$ respectively. The given integral can be expressed as the sum of the integral along L′ and two integrals around the branch cuts and the poles. The integral along L′ diminish as 1/(distance), whereas those around the branch cuts as 1/(distance)^2 and these are generally negligible.

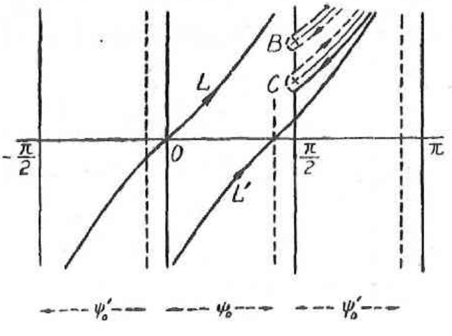

Fig. 3. w-plane.

(iii) $f(w)$ can be expanded as

$$f(w)=f(w_0)+f'(w_0)(w-w_0)+\frac{1}{2}f''(w_0)(w-w_0)^2+\cdots\cdots. \tag{41}$$

As the main part of the integral, except the residues, is contributed from that in the vicinity of the saddle point $w_0=\psi_0$, we can put $w-w_0=\varepsilon\exp(i\pi/4)$ along L′ near it, and put approximately

$$f(w)=-ik_{2,p}\left(r\sin\psi_0+d\cos\psi_0+H\sqrt{\frac{1}{n^2}-\sin^2\psi_0}\right)$$
$$-\frac{k_{2,p}}{2}\left[R_0-\frac{H\left(\frac{1}{n^2}-1\right)\sin^2\psi_0}{\left(\frac{1}{n^2}-\sin^2\psi_0\right)^{3/2}}\right]\varepsilon^2. \tag{42}$$

As

$$\int_{-\infty}^{\infty}\exp\left[-\frac{k_{2,p}}{2}\left\{R_0-\frac{H\left(\frac{1}{n^2}-1\right)\sin^2\psi_0}{\left(\frac{1}{n^2}-\sin^2\psi_0\right)^{3/2}}\right\}\varepsilon^2\right]d\varepsilon$$
$$=\sqrt{\frac{2\pi}{k_{2,p}}}\frac{1}{\sqrt{R_0-\dfrac{H\left(\frac{1}{n^2}-1\right)\sin^2\psi_0}{\left(\frac{1}{n^2}-\sin^2\psi_0\right)^{3/2}}}}, \tag{43}$$

the approximate value of the integral along L′ can be given by

$$\delta_{z,0}=-ik_{2,p}\frac{1}{[R_0]}2\sqrt{1-n^2\sin^2\psi_0}\,G_p'(\psi_0)\exp\left[-ik_{2,p}\left(r\sin\psi_0+d\cos\psi_0\right.\right.$$
$$\left.\left.+H\sqrt{\frac{1}{n^2}-\sin^2\psi_0}\right)\right], \tag{44}$$

where

$$[R_0] = \sqrt{R_0}\ \sqrt{R_0 - \frac{H\left(\frac{1}{n^2}-1\right)\sin^2\psi_0}{\left(\frac{1}{n^2}-\sin^2\psi_0\right)^{3/2}}}\ , \tag{45}$$

$$G_b'(\psi_0) = \frac{6\,m\cos\psi_0\,(2\cos^2\psi_0+1)}{m\sqrt{1-n^2\sin^2\psi_0}\,\left[(2\cos\psi_0+1)^2+4\sin^2\psi_0\cos\psi_0\sqrt{\cos^2\psi_0+2}\right]+9\,n\cos\psi_0}. \tag{46}$$

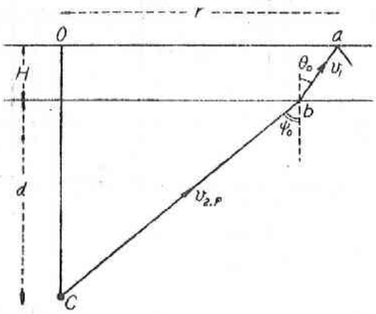

Fig. 4.

Let us consider provisionally that the P waves emitted from the point source are refracted into the water at b on the interface, and arrive at the observing point a on the sea surface, as is shown in Fig. 4, and that the laws of reflection and refraction of the elastic plane waves hold there, then we have

$$\sin\theta_0 = n\sin\psi_0, \tag{47}$$

$$G_p'(\psi_0) = \frac{6\,mn\sqrt{n^2-\sin^2\theta_0}\,(3\,n^2-2\sin^2\theta_0)}{m\cos\theta_0\left[(3n^2-2\sin^2\theta_0)^2+4\sin^2\theta_0\sqrt{n^2-\sin^2\theta_0}\sqrt{3n^2-\sin^2\theta_0}\right]+9n^4\sqrt{n^2-\sin^2\theta_0}}$$

$$= G_p(\theta_0)\,, \tag{48}$$

$$\exp\left[i\,\omega\,t - ik_{2,p}\left(r\sin\psi_0 + d\cos\psi_0 + H\sqrt{\frac{1}{n^2}-\sin^2\psi_0}\right)\right] = \exp\,(i\,\omega\,\tau_0)\,, \tag{49}$$

$$\tau_0 = t - \left(\frac{\overline{cb}}{v_{2,p}} + \frac{\overline{ba}}{v_1}\right), \tag{50}$$

$$R_0 - \frac{H\left(\frac{1}{n^2}-1\right)\sin^2\psi_0}{\left(\frac{1}{n^2}-\sin^2\psi_0\right)^{3/2}} = R_0 - \frac{H(1-n^2)\sin^2\theta_0}{n\cos^3\theta_0} = \overline{cb} + \overline{ba}\left(\frac{n^2-\sin^2\theta_0}{n\cos^2\theta_0}\right), \tag{51}$$

and (44) can be expressed as

$$\delta_{z,0} = -ik_{2,p}\,\frac{1}{[R_0]}\,2\cos\theta_0\,G_p(\theta_0)\exp\,(i\,\omega\,\tau_0)\,. \tag{52}$$

$1/[R_0]$ represents the effect of the geometrical spreading of the rays as the distance from the source increases, $2\cos\theta_0$ expresses the effect of the reflection of the sound waves at the sea surface. $G_p(\theta_0)$ is the refraction coefficient or the ratio of the amplitude of the waves refracted into the water to that of the incident P waves from the solid bottom, when the waves are treated as the plane waves. $\exp\,(i\omega\tau_0)$ is the exact expression of the time factor for the path of waves $\overline{cb}+\overline{ba}$.

When r is very small, the asymptotic expansion (32) can not be used. In the limiting case when $r=0$, $\delta_{z,0}$ becomes

$$\delta_{z,0} = -4\,k_{2,s}^{\,2}\int_0^{\pi/2+i\infty} N\sin w\exp\left[-ik_{2,p}\left(d\cos w + H\sqrt{\frac{1}{n^2}-\sin^2 w}\right)\right]dw\,. \tag{53}$$

The path of integration which starts from the origin $w=0$, and along which the exponential term of the integrand of (53) diminishes most conspicuously, makes the angle $\pi/4$ at the origin. Hence, along the path we can put approximately, in the neighbourhood of the origin

$$\cos w = 1 - i\,\varepsilon\,, \qquad \varepsilon > 0 \tag{54}$$

and

$$\sin w = \sqrt{2i\,\varepsilon}\,, \qquad \sin w\, dw = i d\varepsilon\,. \tag{55}$$

Putting these relations in (53), and integrating from 0 to ∞ with respect to ε, we have

$$\delta_{z,0} = -ik_{2,p}\frac{1}{d+nH}\,2\,\frac{2m}{m+n}\exp\Big[-i\,\omega\Big(\frac{d}{v_{2,p}}+\frac{H}{v_1}\Big)\Big]. \tag{56}$$

(56) is equal to that which can be obtained from (52), when we put $r=\theta_0=\psi_0=0$ in (44) or (52).

(iv) Now let us make some provisional discussion on the integrals around the branch cuts. The major contributins to these integrals come from the neighbourhood of the branch points B and C respectively. Introducing the values of $\cos w$ and $\sin w$ at B and C in $f(w)$ and transforming the expressions adequately, we have to a first approximation a factor

$$\exp\Big[-ik_{2,p}\Big(\frac{1}{n}r-i\sqrt{\frac{1}{n^2}-1}\,d\Big)\Big] = \exp\Big(-i\,\omega\frac{r}{v_1}\Big)\exp\big(-k_1\sqrt{1-n^2}\,d\big) \tag{57}$$

for the integral around B, and a factor

$$\begin{aligned}\exp\Big[-ik_{2,p}\Big(\sqrt{3}\,r-i\sqrt{2}\,d+H\sqrt{\frac{1}{n^2}-3}\Big)\Big] &= \exp\Big[-i\,\omega\Big(\frac{1}{v_{2,p}}\frac{d}{\cos\psi_{0,1}}\\ &+\frac{r-d\tan\psi_{0,1}-H\tan\theta_{0,1}}{v_{2,s}}+\frac{1}{v_1}\frac{H}{\cos\theta_{0,1}}\Big)\Big]\exp\big(-k_{2,p}\sqrt{2}\,d\big)\\ &= \exp\Big[-i\,\omega\Big(\frac{\overline{cb_{1,1}}}{v_{2,p}}+\frac{\overline{b_{1,1}b_{1,2}}}{v_{2,s}}+\frac{\overline{b_{1,2}a}}{v_1}\Big)\Big]\exp\big(-k_{2,p}\sqrt{2}\,d\big)\,,\end{aligned}$$

$$\sin\psi_{0,1} = 1/\sqrt{3}\,,\quad \sin\theta_{0,1} = \sqrt{3}\,n\,. \quad \text{(Fig. 5).} \tag{58}$$

for the integral around C. (57) corresponds to the water waves of R. SATO (1954) which travel along the interface with the velocity of sound in the water. (58) corresponds to the surface S waves of H. NAKANO (1925) or E. R. LAPWOOD (1949) which are propagated along the surface of the semi-infinite elastic solid with the velocity of the S waves. The path corresponding to (58) is illustrated in Fig. 5.

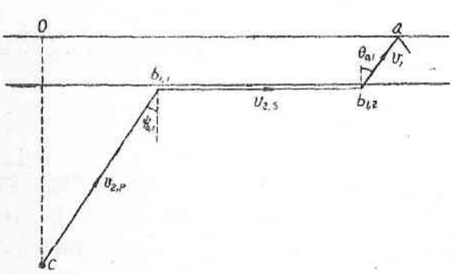

Fig. 5.

(v) The evaluations of $\delta_{z,1}$, $\delta_{z,2}$, $\cdots$ can be carried out quite similarly as in the case of $\delta_{z,0}$. The factor $K(\psi)$ is transformed into $A(\theta)$, when we put $\sin\theta = n\sin\psi$.

$$A(\theta)=\frac{m\cos\theta\left[(3n^2-2\sin^2\theta)^2+4\sin^2\theta\sqrt{n^2-\sin^2\theta}\sqrt{3n^2-\sin^2\theta}\right]-9n^4\sqrt{n^2-\sin^2\theta}}{m\cos\theta\left[(3n^2-2\sin^2\theta)^2+4\sin^2\theta\sqrt{n^2-\sin^2\theta}\sqrt{3n^2-\sin^2\theta}\right]+9n^4\sqrt{n^2-\sin^2\theta}}. \tag{59}$$

$A(\theta)$ is the ratio of the amplitude of the plane sound waves reflected into the water at the surface of the solid bottom to that of the incident plane waves, the angle of incidence being θ.

The results of calculation are

$$\delta_{z,1}=ik_{2,p}\frac{1}{[R_1]}2\cos\theta_1 G_p(\theta_1)A(\theta_1)\exp(i\omega\tau_1), \tag{60}$$

$$\delta_{z,2}=-ik_{2,p}\frac{1}{[R_2]}2\cos\theta_2 G_p(\theta_2)\left[A(\theta_2)\right]^2\exp(i\omega\tau_2), \tag{61}$$

where

$$r=R_1\sin\psi_1,\quad \frac{3H\cos\psi_1}{\sqrt{\frac{1}{n^2}-\sin^2\psi_1}}+d=R_1\cos\psi_1,\quad \sin\theta_1=n\sin\psi_1, \tag{62}$$

$$r=R_2\sin\psi_2,\quad \frac{5H\cos\psi_2}{\sqrt{\frac{1}{n^2}-\sin^2\psi_2}}+d=R_2\cos\psi_2,\quad \sin\theta_2=n\sin\psi_2, \tag{63}$$

$$[R_1]=\sqrt{R_1}\sqrt{R_1-\frac{3H\left(\frac{1}{n^2}-1\right)\sin^2\psi_1}{\left(\frac{1}{n^2}-\sin^2\psi_1\right)^{3/2}}},$$

$$[R_2]=\sqrt{R_2}\sqrt{R_2-\frac{5H\left(\frac{1}{n^2}-1\right)\sin^2\psi_2}{\left(\frac{1}{n^2}-\sin^2\psi_2\right)^{3/2}}}, \tag{64}$$

$$\exp\left[i\omega t-ik_{2,p}\left(r\sin\psi_1+d\cos\psi_1+3H\sqrt{\frac{1}{n^2}-\sin^2\psi_1}\right)\right]=\exp(i\omega\tau_1), \tag{65}$$

$$\exp\left[i\omega t-ik_{2,p}\left(r\sin\psi_2+d\cos\psi_2+5H\sqrt{\frac{1}{n^2}-\sin^2\psi_2}\right)\right]=\exp(i\omega\tau_2), \tag{66}$$

$$\tau_1=t-\left(\frac{\overline{cb_1}}{v_{2,p}}+\frac{3\,\overline{b_1a_1}}{v_1}\right),$$

$$\tau_2=t-\left(\frac{\overline{cb_2}}{v_{2,p}}+\frac{5\,\overline{b_2a_2}}{v_1}\right).\ \text{(Fig. 6)}. \tag{67}$$

The time factors in (60) and (61) correspond to the paths which are illustrated in Fig. 6 (a) and Fig. 6 (b) respectively. $\delta_{z,1}$, $\delta_{z,2}$, $\cdots\cdots$ are due to the sound waves reflected once or twice, $\cdots\cdots$ at the interface.

(vi) As the solutions for the simple harmonic compressional waves originating at a point source within the bottom have been obtained, it remains to obtain the solutions for an impulsive initial disturbance. If the time variation of the potential of the initial disturbance at the source is $\Psi(t)=C/(t^2+c^2)$, C and c being constants, we have to perform the operation

$$\frac{1}{\pi} Re \int_0^\infty d\omega \int_{-\infty}^{\infty} \Psi(\sigma) e^{-i\omega(\sigma-t)} d\sigma , \quad \Psi(\sigma) = \frac{C}{\sigma^2+c^2} \tag{68}$$

to (13), (52), (60) and (61) in order to obtain the expressions D_R, $D_{z,0}$, $D_{z,1}$ and $D_{z,2}$ for the impulsive original shock, corresponding to δ_R, $\delta_{z,0}$, $\delta_{z,1}$ and $\delta_{z,2}$ for the periodic case. The results of calculation are

$$D_R = \frac{2C}{v_{2,p} R} \frac{\tau}{(\tau^2+c^2)^2} , \quad \tau = t - \frac{R}{v_{2,t}} , \tag{69}$$

$$D_{z,0} = \frac{2C}{v_{2,p}} \frac{1}{[R_0]} 2\cos\theta_0 G_p(\theta_0) \frac{\tau_0}{(\tau_0^2+c^2)^2} , \tag{70}$$

$$D_{z,1} = -\frac{2C}{v_{2,p}} \frac{1}{[R_1]} 2\cos\theta_1 G_p(\theta_1) A(\theta_1) \frac{\tau_1}{(\tau_1^2+c^2)^2} , \tag{71}$$

$$D_{z,2} = \frac{2C}{v_{2,p}} \frac{1}{[R_2]} 2\cos\theta_2 G_p(\theta_2) [A(\theta_2)]^2 \frac{\tau_2}{(\tau_2^2+c^2)^2} . \tag{72}$$

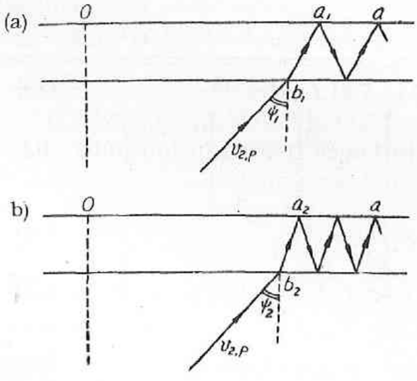

Fig. 6.

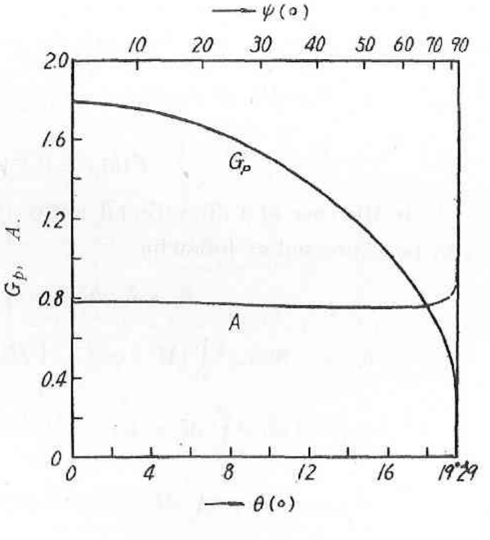

Fig. 7. G_p and A.

Being assumed provisionally that $m=2.5$, $n=1/3$, the values of $G_p(\theta)$ and $A(\theta)$ are shown in Fig. 7. When we assume further that

$$v_1 = 1.50\,\text{km/sec}, \quad v_{2,p} = 4.50\,\text{km/sec}, \quad c = 0.05\,\text{sec}, \quad C = 1\,(\text{km}\cdot\text{sec}),$$
$$d = 10\,\text{km}, \quad H = 1\,\text{km}, \quad r = 5\,\text{km},$$

the results of the numerical calculations of (70), (71) and (72) are shown in Fig. 8.

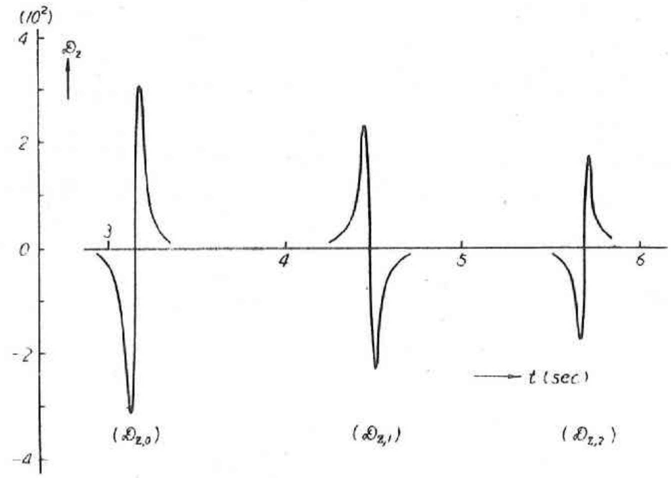

Fig. 8. $D_{z,0}$, $D_{z,1}$ and $D_{z,2}$.

4 The Motion of the Surface of the Sea due to the S Waves emitted from a Point Source beneath the Bottom

(i) The displacement components δ_r and δ_z of the sea surface due to the S waves emitted from a point source beneath the bottom are obtained from (1) and (18) to be

$$\delta_r = 0\,,$$

$$\delta_z = -4\,k_{2,s}{}^2 \int_0^\infty \frac{\eta}{F(k)\cos(\xi H)} \exp(-\zeta\, d)\, J_0(kr) k^3 dk\,. \tag{73}$$

As in the case of a dilatational source of disturbance treated in foregoing chapter, δ_z can be expressed as following :

$$\delta_z = \delta_{z,0} + \delta_{z,1} + \delta_{z,2} + \cdots\cdots\,, \tag{74}$$

$$\delta_{z,0} = -8\,ik_{2,s}{}^2 \int_0^\infty M' \exp(-i\,\xi\, H)\exp(-\zeta\, d)\, J_0(kr)\, k^3\, dk\,, \tag{75}$$

$$\delta_{z,1} = 8\,ik_{2,s}{}^2 \int_0^\infty M' \exp(-3\,i\,\xi\, H)\, K \exp(-\zeta\, d)\, J_0(kr)\, k^3\, dk, \tag{76}$$

$$\delta_{z,2} = -8\,ik_{2,s}{}^2 \int_0^\infty M' \exp(-5\,i\,\xi\, H)\, K^2 \exp(-\zeta\, d)\, J_0(kr)\, k^3\, dk, \tag{77}$$

$$M' = \eta \Big/ \Big[i\,\{4\,k^2\,\eta\zeta - (2\,k^2 - k_{2,s}{}^2)^2\} - \frac{\rho_1}{\rho_2}\frac{\eta}{\xi} k_{2,s}{}^4 \Big]. \tag{78}$$

Putting $k = k_{2,s}\sin w$, we have

$$\xi = k_{2,s}\sqrt{\frac{1}{3\,n^2} - \sin^2 w}\,, \quad \eta = ik_{2,s}\sqrt{\frac{1}{3} - \sin^2 w}\,, \quad \zeta = ik_{2,s}\cos w\,, \tag{79}$$

$$Im\sqrt{\frac{1}{3\,n^2} - \sin^2 w} < 0\,, \quad Im\sqrt{\frac{1}{3} - \sin^2 w} < 0\,, \quad Im\,(\cos w) < 0\,. \tag{80}$$

Using the asymptotic expansion (32), we have

$$\delta_{z,0} = 4\,ik_{2,s}{}^3\sqrt{\frac{2}{\pi k_{2,s} r}}\exp\left(i\frac{\pi}{4}\right)\int_{-\pi/2-i\infty}^{\pi/2+i\infty} N'\sqrt{\sin w}\exp\Big[-ik_{2,s}\Big(r\sin w + d\cos w + H\sqrt{\frac{1}{3n^2}-\sin^2 w}\Big)\Big]dw\,, \tag{81}$$

$$N' = \frac{m\sin^2 w\cos w\sqrt{\frac{1}{3}-\sin^2 w}\;\sqrt{\frac{1}{3n^2}-\sin^2 w}}{m\sqrt{\frac{1}{3n^2}-\sin^2 w}\Big[(2\sin^2 w-1)^2+4\sin^2 w\cos w\sqrt{\frac{1}{3}-\sin^2 w}\Big]+\sqrt{\frac{1}{3}-\sin^2 w}}\,. \tag{82}$$

The factor K becomes

$$K = \frac{m\sqrt{\frac{1}{3n^2}-\sin^2 w}\Big[(2\cos^2 w-1)^2+4\sin^2 w\cos w\sqrt{\frac{1}{3}-\sin^2 w}\Big]-\sqrt{\frac{1}{3}-\sin^2 w}}{m\sqrt{\frac{1}{3n^2}-\sin^2 w}\Big[(2\cos^2 w-1)^2+4\sin^2 w\cos w\sqrt{\frac{1}{3}-\sin^2 w}\Big]+\sqrt{\frac{1}{3}-\sin^2 w}}\,. \tag{83}$$

When r is very small the asymptotic expansion (32) can not be used, but it is clear that $\delta_{z,0}$ is zero in the case $r=0$.

(ii) The integral (81) contains the two valued functions $\lambda_w' = \sqrt{1/3n^2-\sin^2 w}$ and $\lambda_p = \sqrt{1/3-\sin^2 w}$. The points D ($\cos w = -i\sqrt{1/3n^2-1}$, $\sin w = 1/\sqrt{3}\,n$), E ($\cos w = \sqrt{2/3}$, $\sin w = 1/\sqrt{3}$), $\cdots\cdots$ in Fig. 9(a), are the branch points on w-plane. The lines along which $Re(\lambda_w')=0$ and $Re(\lambda_p)=0$ are shown by the solid lines, and those along which $Im(\lambda_w')=0$ and $Im(\lambda_p)=0$ by the broken lines in Fig. 9(b) and Fig. 9(c)

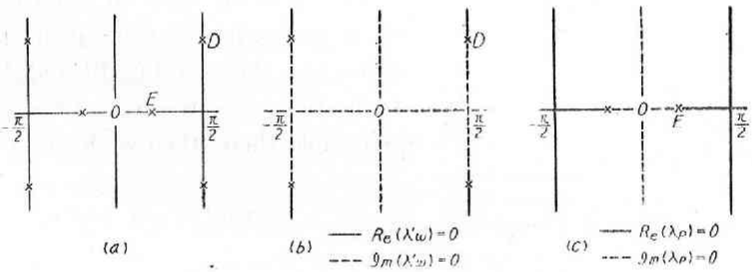

Fig. 9. w-plane.

respectively. We assume that the cuts along which the leaves of the Riemann surface coalesce, are given by $Re(\lambda_w')=0$ and $Re(\lambda_p)=0$. On one of the sheets, on which $Re(\lambda_w')>0$ and $Re(\lambda_p)>0$, we take the given path of integration L ($-\pi/2-i\infty \rightarrow \pi/2+i\infty$). Putting

$$g(w) = -ik_{2,s}\Big(r\sin w + d\cos w + H\sqrt{\frac{1}{3n^2}-\sin^2 w}\Big), \tag{84}$$

we see that the saddle point $w=\chi_0$ is given by

$$r = R_0'\sin\chi_0\,, \quad \frac{H\cos\chi_0}{\sqrt{\frac{1}{3n^2}-\sin^2\chi_0}}+d = R_0'\cos\chi_0\,, \tag{85}$$

and

$$\left(\frac{\partial^2 g}{\partial w^2}\right)_{w=\chi_0} = ik_{2,s}\left[R_0' - \frac{H\left(\frac{1}{3n^2}-1\right)\sin^2\chi_0}{\left(\frac{1}{3n^2}-\sin^2\chi_0\right)^{3/2}}\right]. \tag{86}$$

The integral (81) can be evaluated quite similarly as in foregoing chapter.

$$\delta_{z,0} = -k_{2,s}{}^2\frac{1}{[R_0']}\sin\chi_0\cdot 2\sqrt{1-3n^2\sin^2\chi_0}\;G_s'(\chi_0)\exp\left[-ik_{2\cdot s}\left(r\sin\chi_0+d\cos\chi_0 + H\sqrt{\frac{1}{3n^2}-\sin^2\chi_0}\right)\right], \tag{87}$$

$$[R_0'] = \sqrt{R_0'}\sqrt{R_0' - \frac{H\left(\frac{1}{3n^2}-1\right)\sin^2\chi_0}{\left(\frac{1}{3n^2}-\sin^2\chi_0\right)^{3/2}}}, \tag{88}$$

$$G_s'(\chi_0) = \frac{4}{\sqrt{3}\,n}\times \frac{m\sin\chi_0\cos\chi_0\sqrt{\frac{1}{3}-\sin^2\chi_0}}{m\sqrt{\frac{1}{3n^2}-\sin^2\chi_0}\left[(2\cos^2\chi_0-1)^2+4\sin^2\chi_0\cos\chi_0\sqrt{\frac{1}{3}-\sin^2\chi_0}\right]+\sqrt{\frac{1}{3}-\sin^2\chi_0}}. \tag{89}$$

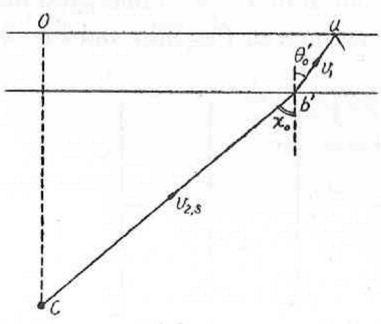

Fig. 10.

Let us consider provisionally that the S waves emitted from the point source c are refracted into the water at b' on the interface, and arrive at the observing point a on the sea surface as is shown in Fig. 10, and that the laws of reflection and refraction of the plane elastic waves hold there, then we have

$$\sin\theta_0' = \sqrt{3}\,n\sin\chi_0, \tag{90}$$

$$G_s'(\chi_0) = \frac{4\sqrt{3}\,mn\sin\theta_0'\sqrt{n^2-\sin^2\theta_0'}\sqrt{3n^2-\sin^2\theta_0'}}{m\cos\theta_0'\left[(3n^2-2\sin^2\theta_0')^2+4\sin^2\theta_0'\sqrt{n^2-\sin^2\theta_0'}\sqrt{3n^2-\sin^2\theta_0'}\right]+9n^4\sqrt{n^2-\sin^2\theta_0'}} = G_s(\theta_0'), \tag{91}$$

$$\exp\left[i\omega t - ik_{2,s}\left(r\sin\chi_0+d\cos\chi_0+H\sqrt{\frac{1}{3n^2}-\sin^2\chi_0}\right)\right] = \exp(i\omega\tau_0'), \tag{92}$$

$$\tau_0' = t-\left(\frac{\overline{cb'}}{v_{2,s}}+\frac{\overline{b'a}}{v_1}\right). \tag{93}$$

(87) becomes

$$\delta_{z\cdot 0} = -k_{2,s}{}^2\frac{1}{[R_0']}\sin\chi_0\cdot 2\cos\theta_0'\,G_s(\theta_0')\exp(i\omega\tau_0'). \tag{94}$$

The factor $\sin\chi_0$ denotes the variation of the amplitude of the S waves emitted from the point source with the angle χ_0, and $G_s(\theta_0')$ is the ratio of the amplitude of the sound waves refracted into the water to that of the incident S waves from the solid bottom, when the waves are treated as the plane waves.

(iii) Now let us discuss somewhat on the integrals around the branch point. Putting the values of $\cos w$ and $\sin w$ at D and E in $g(w)$, we have to a first approximation, a factor

$$\exp\left[-ik_{2,s}\left(\frac{1}{\sqrt{3}\,n}r-i\sqrt{\frac{1}{3n^2}-1}\,d\right)\right]=\exp\left(-i\omega\frac{r}{v_1}\right)\exp\left(-k_1\sqrt{1-3n^2}\,d\right) \quad (95)$$

for the integral around D, and a factor

$$\begin{aligned}&\exp\left[-ik_{2,s}\left(\frac{1}{\sqrt{3}}r+\sqrt{\frac{2}{3}}\,d+H\sqrt{\frac{1}{3n^2}-\frac{1}{3}}\right)\right]\\&=\exp\left[-i\omega\left(\frac{1}{v_{2,s}}\frac{d}{\cos\chi_{0,1}}+\frac{r-d\tan\chi_{0,1}-H\tan\theta_{0,1}'}{v_{2,p}}+\frac{1}{v_1}\frac{H}{\cos\theta_{0,1}'}\right)\right]\\&=\exp\left[-i\omega\left(\frac{\overline{cb_{1,1}'}}{v_{2,s}}+\frac{\overline{b_{1,1}'b_{1,2}'}}{v_{2,p}}+\frac{\overline{b_{1,2}'a}}{v_1}\right)\right],\ \sin\chi_{0,1}=\frac{1}{\sqrt{3}},\ \sin\theta_{0,1}'=\sqrt{n} \end{aligned}\quad (96)$$

for the integral around E. (95) corresponds to the water waves travelling along the interface with the velocity of sound in the water, and (96) corresponds to the surface P waves which is propagated along the surface of the semi-infinite elastic solid with the velocity of the P waves.

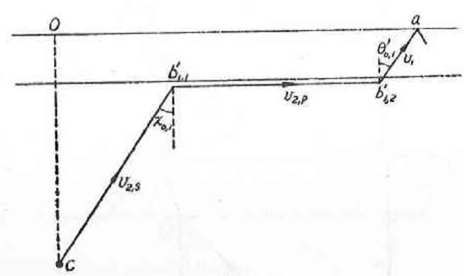

Fig. 11.

(iv) The evaluation of $\delta_{z,1}$, $\delta_{z,2}$, ... can be carried out similarly as in the case of $\delta_{z,0}$. The factor $K(\chi)$ is transformed into $A(\theta)$ given as (59), when we put $\sin\theta=\sqrt{3}\,n\sin\chi$. The results of calculation are:

$$\delta_{z,1}=k_{2,s}{}^2\frac{1}{[R_1']}\sin\chi_1\cdot 2\cos\theta_1'\,G_s(\theta_1')\,A(\theta_1')\exp\left(i\omega\tau_1'\right), \quad (97)$$

$$\delta_{z,2}=-k_{2,s}{}^2\frac{1}{[R_2']}\sin\chi_2\cdot 2\cos\theta_2'\,G_s(\theta_2')\,[A(\theta_2')]^2\exp\left(i\omega\tau_2'\right), \quad (98)$$

where

$$r=R_1'\sin\chi_1,\quad \frac{3H\sin\chi_1}{\sqrt{\dfrac{1}{3n^2}-\sin^2\chi_1}}+d=R_1'\cos\chi_1,\quad \sin\theta_1'=\sqrt{3}\,n\sin\chi_1, \quad (99)$$

$$r=R_2'\sin\chi_2,\quad \frac{5H\sin\chi_2}{\sqrt{\dfrac{1}{3n^2}-\sin^2\chi_2}}+d=R_2'\cos\chi_2,\quad \sin\theta_2'=\sqrt{3}\,n\sin\chi_2, \quad (100)$$

$$[R_1'] = \sqrt{R_1'}\sqrt{R_1' - \frac{3H\left(\frac{1}{3n^2}-1\right)\sin^2\chi_1}{\left(\frac{1}{3n^2}-\sin^2\chi_1\right)^{3/2}}}\,,$$

$$[R_2'] = \sqrt{R_2'}\sqrt{R_2' - \frac{5H\left(\frac{1}{3n^2}-1\right)\sin^2\chi_2}{\left(\frac{1}{3n^2}-\sin^2\chi_2\right)^{3/2}}}\,. \tag{101}$$

$$\exp\left[i\omega t - \left(r\sin\chi_1 + d\cos\chi_1 + 3H\sqrt{\frac{1}{3n^2}-\sin^2\chi_1}\right)\right] = \exp(i\omega\tau_1')\,, \tag{102}$$

$$\exp\left[i\omega t - \left(r\sin\chi_2 + d\cos\chi_2 + 5H\sqrt{\frac{1}{3n^2}-\sin^2\chi_2}\right)\right] = \exp(i\omega\tau_2')\,, \tag{103}$$

$$\tau_1' = t - \left(\frac{\overline{cb_1'}}{v_{2,s}} + \frac{3\overline{b_1'a_1'}}{v_1}\right), \quad \tau_2' = t - \left(\frac{\overline{cb_2'}}{v_{2,s}} + \frac{5\overline{b_2'a'_2}}{v_1}\right). \quad \text{(Fig. 12)} \tag{104}$$

The time factors in (97) and (98) correspond to the paths illustrated in Fig. 12 (a) and Fig. 12 (b) respectively.

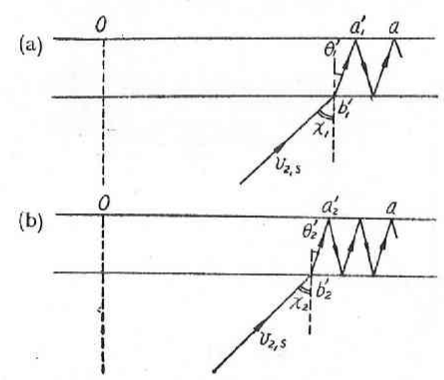

Fig. 12.

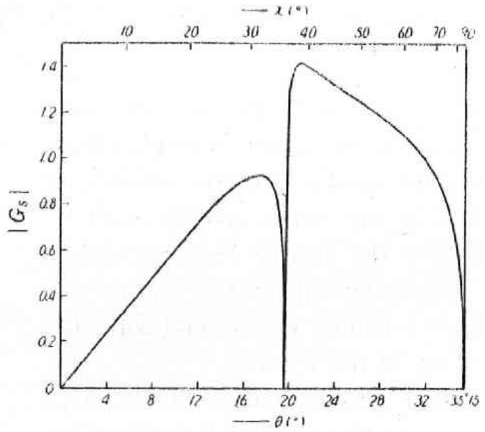

Fig. 13. $|G_s|$

(v) When we perform the operation (68) to (19), (94), (97) and (98), we have D_χ, $D_{z,0}$, $D_{z,1}$ and $D_{z,2}$ for the aperiodic case of shock type, corresponding to δ_χ, $\delta_{z,0}$, $\delta_{z,1}$ and $\delta_{z,2}$ for the periodic case. It must be noticed that while $\sin\theta$ is less than $\sqrt{3}\,n$ it becomes greater than n if $\sin\chi > 1/\sqrt{3}$, and then $G_s(\theta)$ and $A(\theta)$ become complex quantities.

As far as $\sin\theta < n$, $G_s(\theta)$ and $A(\theta)$ are real, and the results of operation of (68) are

$$D_\chi = \frac{2C'}{v_{2,s}^2}\frac{1}{R}\sin\chi\frac{3\tau^2-c^2}{(\tau^2+c^2)^3}\,, \quad \Psi = \frac{C'}{t^2+c^2}\,, \quad \tau = t - \frac{R}{v_{2,s}} \tag{105}$$

$$D_{z,0} = \frac{2C'}{v_{2,s}^2}\frac{1}{[R_0']}\sin\chi_0\cdot 2\cos\theta_0'\,G_s(\theta_0')\frac{3\tau_0^2-c^2}{(\tau_0^2+c^2)^3}\,, \tag{106}$$

$$D_{z,1} = -\frac{2C'}{v_{2,s}^2}\frac{1}{[R_1']}\sin\chi_1\cdot 2\cos\theta_1'\,G_s(\theta_1')\,A(\theta_1')\frac{3\tau_1^2-c^2}{(\tau_1^2+c^2)^3}\,, \tag{107}$$

$$D_{z,2}=\frac{2C'}{v_{2,s}}\frac{1}{[R_2']}\sin\chi_2\cdot 2\cos\theta_2'\,G_s(\theta_2')\,[A(\theta_2')]^2\,\frac{3\,\tau_2^2-c^2}{(\tau_2^2+c^2)^3}\,. \tag{108}$$

When $\sin\theta>n$, we have, as $\sqrt{n^2-\sin^2\theta}=-i\sqrt{\sin^2\theta-n^2}$,

$$G_s(\theta)=|G_s(\theta)|\exp\left(i\,\alpha_1-i\frac{\pi}{2}\right), \tag{109}$$

$$|G_s(\theta)|=\frac{4\sqrt{3}\,mn\sin\theta\sqrt{\sin^2\theta-n^2}\sqrt{3n^2-\sin^2\theta}}{\left[m^2\cos^2\theta\,(3n^2-\sin^2\theta)^4+(4m\sin^2\theta\cos\theta\sqrt{3n^2-\sin^2\theta}+9\,n^4)^2\,(\sin^2\theta-n^2)\right]^{1/2}}, \tag{110}$$

$$\tan\alpha_1=\frac{(4\,m\sin^2\theta\cos\theta\sqrt{3\,n^2-\sin^2\theta}+9\,n^4)\sqrt{\sin^2\theta-n^2}}{m\cos\theta\,(3\,n^2-\sin^2\theta)^2}, \tag{111}$$

$$A(\theta)=|A(\theta)|\exp\left(-i\alpha_2+i\beta_2\right)$$

$$|A(\theta)|=\left[\frac{m^2\cos^2\theta\,(3n^2-2\sin^2\theta)^4+\{4\,m\sin^2\theta\cos\theta\sqrt{\sin^2\theta-n^2}\sqrt{3n^2-\sin^2\theta}-9n^4\sqrt{\sin^2\theta-n^2}\}^2}{m^2\cos^2\theta\,(3n^2-2\sin^2\theta)^4+\{4\,m\sin^2\theta\cos\theta\sqrt{\sin^2\theta-n^2}\sqrt{3n^2-\sin^2\theta}+9n^4\sqrt{\sin^2\theta-n^2}\}^2}\right]^{1/2}, \tag{112}$$

$$\tan\binom{\alpha_2}{\beta_2}=\frac{4\,m\sin^2\theta\cos\theta\sqrt{\sin^2\theta-n^2}\sqrt{3n^2-\sin^2\theta}\mp 9n^4\sqrt{\sin^2\theta-n^2}}{m\cos\theta\,(3n^2-2\sin^2\theta)^2}, \tag{113}$$

$$D_{z,0}=\frac{2C'}{v_{2,s}^2}\frac{1}{[R_0']}\sin\chi_0\cdot 2\cos\theta_0'\,|G_s(\theta_0')|\,\frac{1}{c}\,\frac{(3\,\tau_0^2-c^2)\,c\sin\alpha_1+(\tau_0^2-3\,c^2)\tau_0\cos\alpha_1}{(\tau_0^2+c^2)^3}, \tag{114}$$

$$D_{z,1}=-\frac{2C'}{v_{2,s}^2}\frac{1}{[R_1']}\sin\chi_1\cdot 2\cos\theta_1'\,|G_s(\theta_1')|\,|A(\theta_1')|$$
$$\times\frac{1}{c}\,\frac{(3\,\tau_1^2-c^2)\,c\sin(\alpha_1-\alpha_2+\beta_2)+(\tau_1^2-3\,c^2)\,\tau_1\cos(\alpha_1-\alpha_2+\beta_2)}{(\tau_1^2+c^2)^3}, \tag{115}$$

....

The values of $|G_s(\theta)|$ and $D_{z,0}$, $D_{z,1}$ and $D_{z,2}$ for the numerical values of quantities given in foregiong chapter, are illustrated in Fig. 13 and Fig. 14. In this case $\sin\theta$ is less than n. There exist no direct relations between the motions illustrated in Fig. 8 and Fig. 14.

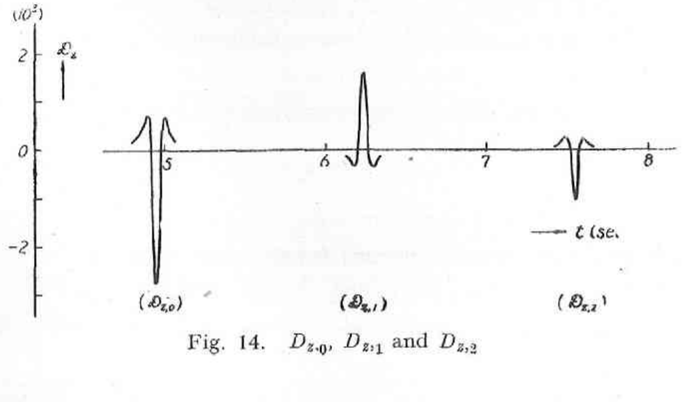

Fig. 14. $D_{z,0}$, $D_{z,1}$ and $D_{z,2}$

5 Summary

The propagation of the sound waves in the sea of uniform depth overlying the solid bottom, due to the dilatational and distortional waves emitted from the point source beneath the bottom, is investigated theoretically. Especially the solutions for the principal motion of the sea surface not very far from the epicenter are obtained, the values of the integrals being evaluated by the method of the steepest descent. The motions of the sea surface due to the impulsive sound waves refracted into the sea and reflected at the sea bottom once or twice are illustrated for some special cases. The results of the calculation may be useful for the interpretation of the phenomena of sea shocks.

References

HONDA, H. and NAKAMURA, K.: "On the Reflection and Refraction of the Explosive Sounds at the Ocean Bottom". *Science Rep. Tôhoku Univ. Ser.* 5, **4**, 125-133, **6**, 70–84, (1952, 1954).

LAMB, H.: "On the Propagation of Tremors over the Surface of an Elastic Solid". *Phil. Trans. Roy. Soc. London*, A. **203**, 1-42, (1904).

LAPWOOD, E. R.: "The Disturbance due to a Line Source in a Semi-infinite Elastic Medium." *ibid.*, A. **841**, 63-100, (1949).

NAKANO, H.: "On Rayleigh Wave," *Jap. Journ. Astro. and Geophys.*, **2**, 1–94, (1925).

OFFICER, C. B.: "The Refraction Arrival in Water Covered Areas". *Geophysics*, **13**, 805-819, (1953).

PEKERIS, C. L.: "Theory of Propagation of Explosive Sound in Shallow Water". *Geol. Soc. Amer. Memoir*, **27**, 1–117, (1948).

PRESS, F. and EWING, M.: "A theory of Microseisms with Geologic Applications". *Trans. Amer. Geophys. Union*, **29**, 163-174, (1948).

PRESS, F. and EWING, M.: "Propagation of Explosive Sound in a Liquid Layer overlying a Semi-infinite Elastic Solid". *Geophysics*, **15**, 426-446, (1950).

PRESS, F., EWING, M. and TOLSTOY, I.: "The Airy Phase of Shallow-Focus Submarine Earthquakes". *Bull. Seis. Soc. Amer.*, **40**, 111-148, (1950).

SATO, R.: "On the Propagation of Tremors along the Interface between Solid and Water produced by a Point Source in a Solid". Zisin. **7**, 8–20, 77–88, (1954).